View Point

Bifurcation Paradigm
- A Short Summary of Discussion at the Workshop on "Self-Organization and Transport in Electromagnetic Turbulence"

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Recent experimental work on magnetic fusion devices has uncovered many examples of threshold phenomena where nonlinear relaxation process with following characteristic features are observed:

- Slow increase of a mean parameter followed by its sudden fall, in the form of a burst;
- Recovery from the burst in a short duration and recurrence of the burst cycle at an average frequency determined by the input source as the key controlling parameter;
- Probabilistic nature of time series with properties determined by 'distance' from threshold;
- frequent occurrence of 'precursor' which indicates the crash by sudden onset of a symmetry breaking perturbation called 'trigger mode'.

Table 1 presents a list of such phenomena. A somewhat detailed review of such collapse events is given in [1].

In this communication, we wish to emphasize that the conventional paradigm of such phenomena in terms of a continuous transition through a linear instability and associated nonlinear effects has outlived its utility. One specific instance of a conventional model is that for m = 1 saw-tooth events discussed in [2]. The linear growth rate γ_L is expressed in terms of global plasma parameters. A change of γ_L is then induced by the evolution of global parameters, viz.,

$$\frac{\partial \gamma_L}{\partial t} = \frac{\partial \gamma_L}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial \gamma_L}{\partial \beta}\frac{\partial \beta}{\partial t} + \cdots$$

where q and β are respectively the safety factor and the normalized plasma pressure. Note that $\partial\gamma_L/\partial q$ and $\partial\gamma_L/\partial\beta$ are regular mathematical functions. Since the time evolution $\partial q/\partial t$, $\partial\beta/\partial t$, etc. induces changes over the transport time scale, a sudden onset of crash cannot be described by the conventional view.

An alternative way of understanding these nonlinear relaxation processes is to model them as threshold bifurcation phenomena. An essential element of such models is a hysteresis in the dynamics of the symmetry-breaking perturbations that induce the transport of average plasma quantities. A characteristic equation for such a model may be written as [3]:

$$\frac{\partial}{\partial t} T = \frac{\partial}{\partial x}\left(\chi[T, \cdots]\frac{\partial}{\partial x} T\right) + S$$

in which the coefficient $\chi[T, \cdots]$, being a functional of plasma parameters, has a hysteresis nature (Fig.1). A global plasma parameter T has been chosen as an

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Table 1 List of relaxation oscillation. Lower list are for those with similar characteristics but is too catastrophic to be repeated.

Name	device	Observed response	Trigger mode	Time scale of trigger
Sawtooth	ALL	T_e	$m/n = 1/1$ (etc. ?)	< 100 μs
Internal collapse (Heliotron)	Heliotron-E	T_e	$m/n = 2/1$	< 100 μs
ELMs (Type-I)	ALL	T_e	$n \gg 1$ (ballooning, peeling?)	10 – 100 μs
ELMs (Type-I)	ALL	T_e	$n \gg 1$	↑
ELMs (Dither)	ALL	Barrier	—	↑
MTE	D-III D	V'_ϕ	$n \gg 1$	< 100 μs
IRE	Spherical Tokamak	ℓ_i	$m/n = 1/1$	~ 100 μs
Electric Pulsation	CHS	E_r	—	a few 10 μs
ITB Pulsation	W7-AS	Barrier	—	10 – 100 μs
Monster sawtooth	JET	T_e	$m/n = 1/1$	< 100 μs
High-β collapse	~ ALL	$\beta(0)$	$m/n = 1/1$	< 100 μs
disruption (thermal quench)	ALL	T_e	$m/n = 1/1$	10 – 100 μs
minor disruption	~ ALL	T_e		
X-event	JET	χ_i	—	< 100 μs

Abbreviations: ELMs (edge-localized modes), MTE (momentum transfer events), BLM (barrier-localized mode), IRE (internal reconnection events).
In this table, 'ALL' means that the phenomena has been observed in majority of tokamaks. (Based on [1].)

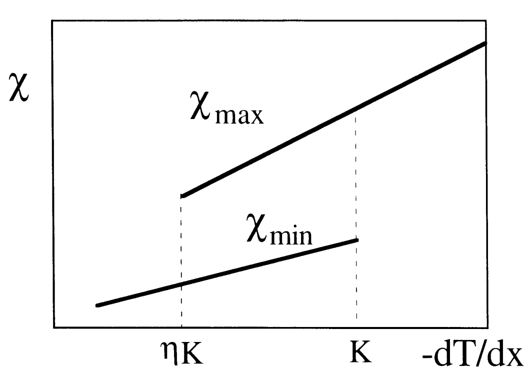

Fig. 1 Hysteresis in the transport coefficient of global plasma parameters.

example and can readily be extended to a set of plasma parameters (T, n, E_r, $\cdots$). The above description allows us to model relaxation phenomena with collapse, because of the introduction of hysteresis effects and is appropriate when the onset-time (i.e., the growth time of trigger mode activity) is much smaller than the duration of the burst. If we want to include effects due to dynamics of trigger mode, we may use the method of time-scale separation. As an example, one has the combined set of equations [4]:

$$\frac{\partial}{\partial t} T = \frac{\partial}{\partial x} \left(\chi \frac{\partial}{\partial x} T \right) + S$$

$$\frac{\partial}{\partial t} \chi + \frac{\chi}{\tau} = \frac{Q}{\tau}$$

$$Q = \left\{ \begin{array}{ll} \chi_{\max} & \text{if } \left| \partial T / \partial x \right| > K \\ \chi_{\min} & \text{if } \left| \partial T / \partial x \right| < \eta K \end{array} \right\}$$

where time and length are normalized to a global time and length, respectively. This model set of equations has minimal features:

- Two parameters - the global one (T) and the other determined by fluctuations (χ) - coupled to each other;
- Three different time scales, viz., $\chi_{\min}^{-1}$ (related to the slow evolution in absence of source), τ (onset rate of trigger mode), and $\chi_{\max}^{-1}$ (related to the fast crash);
- Threshold K;
- Hysteresis through parameter $\eta < 1$
- Propagation like avalanches after the onset of collapse.

The above approach based on bifurcation and hysteresis is a powerful method to study relaxation/ transition phenomena in plasma physics. This belongs to a class of time-dependent Ginzburg-Landau equations which are extremely useful in physics to describe transition phenomena in time and phase interfaces in space. Modelling efforts in the magnetic confinement theories using these methods include examples like Type-I ELMs, dithering ELMs, Sawtooth, statistical excitations, burst transport, etc. [5]

Here we have presented the model equations in an inductive way. A deductive analysis has been made in some cases, for example one may note the discussion on M-mode transition [6] (which is a transition from electrostatic to electromagnetic turbulence with magnetic braiding). One has, however, a confidence that the hysteresis of the transport equation can be derived by deductive method in more general circumstances. This is because many theoretical calculations have shown the existence of nonlinear instabilities [7]. Such instabilities are excited because of finite amplitude effects, in a parameter space where linear theory predicts stability.

The above description is the simplest picture. As mentioned before, for realistic systems more degrees of freedom might be necessary in the fluctuations, as well as in the global parameters. One example is that of Type I ELMs, where it is believed that the peeling mode and ballooning mode are jointly determining the dynamics [8]. Another example is that in which high-n ballooning mode and $m/n = 1/1$ mode are believed to cooperate in producing the high-β collapse. In these cases, the number of global parameters may also be greater that one (e.g., in the Type-I ELMs problem, the edge pressure gradient and the edge current density might both be important control parameters); Associated with multi-parameter problem, a higher-order hysteresis is necessary. Moreover, ultimately these problems will have to be treated with methods of statistical physics. Thus, onset of transition in the presence of 'noise' or for systems with 'diffused' critical parameter will become an important issue.

In summary, a bifurcation paradigm is presented to analyze the relaxation phenomena in plasmas. Experimentalists are encouraged to move away from the conventional paradigm towards the bifurcation picture, and to look for characteristic signatures. These are, multiple time scales (e.g., build-up time, growth time of the trigger mode, crash time of a burst), measurement of hysteresis effects, probabilistic characterization of time series, evidence of propagating fronts, etc.

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