

## Multi-Ion-Species Effects on Power Spectra and Autocorrelation Functions of Ion Bernstein Waves

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Ion Bernstein waves in plasmas containing multiple ion species are studied numerically. In thermal equilibrium plasmas, a great number of Bernstein waves are excited at the harmonics of ion cyclotron frequencies. The autocorrelation function of the quasi-mode consisting of these waves is strongly damped and does not recover to its initial value, even if the abundances of heavy ions are very small.

### Keywords:

multi-ion-species plasma, ion Bernstein waves, phase mixing, wave damping

It is generally thought that waves propagating perpendicular to a magnetic field in a collisionless plasma are not damped. It has been shown, however, that a perpendicular magnetosonic pulse can be damped in a multi-ion-species plasma, owing to the heavy ion acceleration [1].

The damping of perpendicular electrostatic waves has also been studied by theory and simulation [2,3]. In the limit as the magnetic field approaches zero, the electron Bernstein waves collectively act as a single damped mode, although each Bernstein wave with  $\omega \simeq n\Omega_e$  is undamped [4], where  $\Omega_e$  is the electron cyclotron frequency and  $n$  the integer. The damping is caused by the phase mixing of an infinite set of closely spaced real frequencies. When the magnetic field is finite, the quasi-mode consisting of these waves shows periodic behavior with a time period  $2\pi/|\Omega_e|$ . Recurrence is expected to also occur in ion Bernstein waves in a single-ion-species plasma. The recurrence time will be the ion cyclotron period  $2\pi/\Omega_i$ .

Space plasmas such as the solar corona, however, have many ion species. Moreover, each ion species has many different ionic charge states. Numerous different ion cyclotron frequencies and their harmonics will thus exist in those plasmas. The collective behavior of Bernstein waves could then be quite different from that in a single-ion-species plasma with one ionic charge state.

In this paper, we study the collective behavior of ion Bernstein waves in a multi-ion-species plasma. It is found that the autocorrelation function of the quasi-mode is initially damped owing to the phase mixing. Furthermore, the recurrence is incomplete; in a practical situation, the amplitude will not recover if the plasma has many ion species.

In a thermal equilibrium plasma, the fluctuation spectrum of ion Bernstein waves is given as

$$\frac{|E_{k,\omega}|^2}{8\pi} = \sum_n \frac{\pi k_B T \delta(\omega - \omega_n)}{\omega \partial \epsilon(k, \omega) / \partial \omega |_{\omega = \omega_n}}. \quad (1)$$

Here,  $k_B$  is the Boltzmann constant, and  $\omega_{in}$  is the frequency of the  $n$ th harmonic Bernstein wave for ion species  $i$ . The dielectric function  $\epsilon(k, \omega)$  is written as

$$\epsilon = 1 + \sum_j \frac{k_{Dj}^2}{k^2} \left[ 1 - \sum_n \frac{\omega I_n(\mu_j) e^{-\mu_j}}{\omega - n\Omega_j} \right], \quad (2)$$

where the subscript  $j$  refers to ion species (H, He, C, etc.) or electrons ( $j=e$ ), and  $k_{Dj}$  is the Debye wavenumber. Also,  $I_n$  is the modified Bessel function of the  $n$ th order with  $\mu_j$  being  $k^2 \rho_j^2$ , where  $\rho_j$  is the gyroradius. We study the collective behavior of many Bernstein waves for the wavenumber  $k$ . The autocorrelation function  $C_k(\tau)$  of the quasi-mode is obtained from the fluctuation spectrum  $|E_{k,\omega}|^2$  through the

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Fourier transformation in  $\omega$  as

$$C_k(\tau) = \sum_{i,n} \frac{\exp(-i\omega_{in}\tau)}{\omega \partial \mathcal{E}(k, \omega) / \partial \omega|_{\omega=\omega_{in}}}, \quad (3)$$

which is normalized to  $C_k(0)$ .

We numerically calculate  $|E_{k,\omega}|^2$  for low-frequency fluctuation with  $\omega \ll |\Omega_e|$ , retaining terms from  $n = -30$  to 30 for ions and the  $n = 0$  term for electrons in Eq. (2). We then obtain  $C_k(\tau)$ . We discuss in this way three different plasmas; single-ion (H), two-ion (H and He), and six-ion (H, He, C, O, Fe, and Si) species plasmas. The cyclotron frequencies of these ions normalized to  $\Omega_H$  are taken to be  $\Omega_{He} = 0.5$ ,  $\Omega_C = 0.417$ ,  $\Omega_O = 0.375$ ,  $\Omega_{Si} = 0.321$ , and  $\Omega_{Fe} = 0.232$ . All the ion species have equal temperature. In the two-ion-species plasma, the density of He normalized to that of H is  $n_{He} = 0.1$ . In the six-ion-species plasma, the heavy-ion densities are  $n_{He} = 0.1$ ,  $n_C = n_O = 0.01$ , and  $n_{Si} = n_{Fe} = 0.005$ . The magnetic field strength is  $|\Omega_e|/\omega_{pe} = 20$ .

Figure 1 shows frequency spectra for  $k\rho_H = 8$  in the three plasmas. The spectra  $P(\omega)$  are normalized to  $\pi k_B T$ . In the single-ion-species plasma, the peaks are at  $\omega \approx n\Omega_H$ , which correspond to the frequencies of H Bernstein modes. In the two-ion-species plasma, besides these modes, He Bernstein modes are excited at  $\omega \approx n\Omega_{He}$ . In the six-ion-species plasma, a great number of modes exist in the low frequency region  $\omega/\Omega_H < 5$ , because Bernstein modes for heavy ions are also excited. Although the abundances of heavy ions are small, the power spectrum is significantly changed.

Figure 2 shows time variations of autocorrelation functions normalized to their initial values  $C_k(0)$ . In the single-ion-species plasma, the periodic behavior with time period  $2\pi/\Omega_H$  is observed; the amplitude is initially damped due to phase mixing of many harmonic modes with  $\omega \approx n\Omega_H$ , and it is recovered to the initial value because of the phase coherence reestablished at times  $\Omega_H \tau \approx 2n\pi$ . The recurrence peak values are almost the same as the initial one. In the two-ion-species plasma, periodic behavior is also observed. In the six-ion-species plasma, the recurrence peaks are substantially reduced. The amplitude does not recovered to its initial value; this is because the phase coherence among many modes shown in the bottom panel of Fig. 1 has not been established by  $\Omega_H t = 45$ .

If we observe the correlation function for a much longer time, the recurrence peak might return to the initial value. However, for example, in the solar corona, the number of ion species is much greater than six, and many mode frequencies are almost infinitely closely

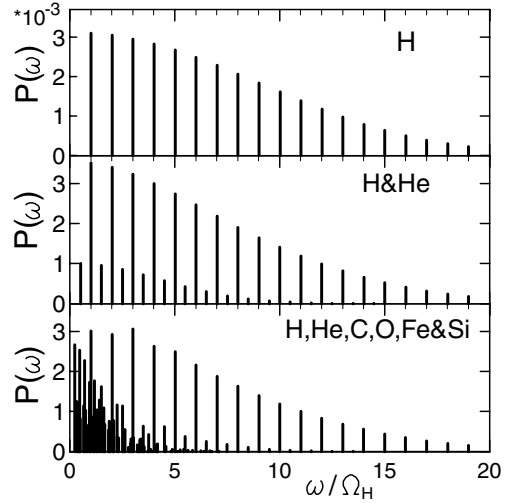


Fig. 1 Frequency spectra in three plasmas.

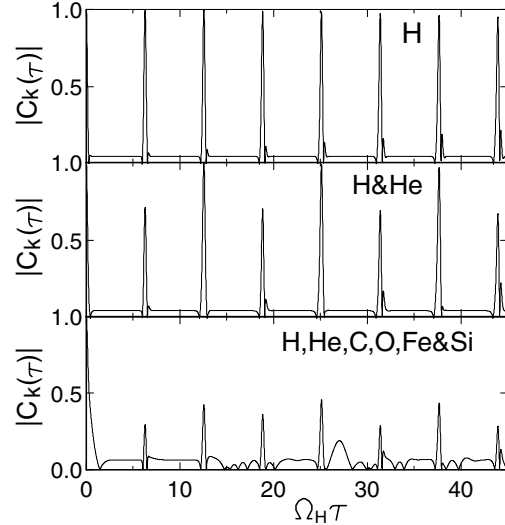


Fig. 2 Autocorrelation functions.

spaced in the low frequency region. We thus expect that the recurrence peaks would never return to the initial value, even though the abundances of heavy ions are small. This damping mechanism would play an important role in space plasmas.

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