δf Simulation and the Radial Electric Field

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The neoclassical radial electric fields in a tokamak are calculated for the shifted Maxwellian distribution function by using a δf Monte Carlo particle simulation code "FORTEC-FSM". The calculation results agree well with the analytical estimations. The present method will be useful for the determination of the neoclassical radial electric field in tokamak plasmas with input momentum or torque by, e.g., NBI heating.

Keywords: radial electric field, δf simulation, tokamak, shifted-Maxwellian

Many aspects of plasma transport such as bootstrap current, electric conductivity, and plasma flow can be explained by standard neoclasical theory, which is constructed under the conditions of $\rho_p \ll L$ and $M_p \ll 1$ (ρ_p is the poloidal Larmor radius, L is the plasma characteristic length, and M_p is the poloidal Mach number). However, these assumptions break down near the magnetic axis because of complicated particle orbits different from standard banana and passing particle orbits [1]. In the plasma with steep ∇p (p is the plasma pressure) as seen in the ETB (edge transport barrier) and ITB (internal transport barrier) discharges, standard neoclassical theory does not hold since it does not include the effects of finite orbit width (FOW). To take the FOW effects into account, δf Monte Calro particle simulations have been develpoed [2,3]. These simulations clarified new physics which can not be contained in the standard neoclasical theory in the limit of small orbit width (SOW). Further, the self-consistent radial electric field has been successfully calculated in a tokamak using a low-noise δf simulation code "FORTEC-E" in both small and large ∇p plasmas [4]. The calculation was done in this paper for the plasma with no toroidal rotations or torque assuming constant q (q is the safety factor).

In the present paper, we calculate the radial electric field for the plasma having local shifted Maxwellian velocity distributions in a tokamak. This situation corresponds to the plasma with toroidal rotation or torque externally induced by, e.g. NBI heating. The calculation will be carried out by using the δf code "FORTEC-FSM", in which many ions are loaded initially according to the local shifted Maxwellian distribution. The calculation results are compared with analytical estimation.

Only ions are treated in the present paper and the density n, ion temperature T, and the plasma parallel flow velocity $u_{\|}$ are assumed to be functions of position r and fixed with time. On the other hand, the electric potential Φ is the function of position r and time t. The drift kinetic equation for ion distribution function is given, in $(\varepsilon, \mu, \vec{x})$ space, by

$$\frac{\partial f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f}{\partial \varepsilon} + \left(\vec{v_{\|}} + \vec{v_d}\right) \cdot \nabla f = C(f,f) \quad (1)$$

The notations are standard. This drift kinetic equation is solved by the δf code "FORTEC-FSM" which employs the two weighting scheme with accurate linear collision operator [3] and the variance reduction method for weight spreading [5]. The shifted Maxwellian distribution function has a form of

$$f_{SM} = e^{e\Phi/T}\frac{n}{\left(\pi v_{th}^2\right)^{3/2}} e^{-m(\varepsilon - u_{\|} v_{\|} + u_{\|}^2/2)/T} \qquad (2)$$

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We consider a large aspect ratio tokamak, in which $B_t = B_0(1-(r/R)\cos\theta)$ and $B_p = rB_0/(qR)$, where B_t and B_p are toroidal and poloidal magnetic field, respectively and q is the safety factor. It should be noted that $v_{||}$ is a function of ε, μ, r, θ and t.

Only ion current is considered in the polarization and diffusion processes since electron current is negligibly small in these processes. The equation for the radial electric field becomes (cgs-unit)

$$\left(1+\frac{c^2}{v_A^2}\right)\frac{\partial E_r}{\partial t} = -4\pi e\Gamma_i \quad (3)$$

$$\Gamma_i = \left\langle \int d^3v v_{dr} f \right\rangle \quad (4)$$

Here, v_{dr} is the radial drift velocity and Γ_i is the flux-averaged particle flux due to ion-ion collisions, which is generated by the FOW effect even if ∇p is very small. Eq.(3) means that the radial electric field is generated so as to vanish the particle flux due to like particle collisions. The particle flux is calculated in "FORTEC-FSM" in each cell devided in the r-direction. Eq.(3) is solved with the time step less than the GAM (geodesic acoustic mode) period ω_{GAM}^{-1} [6].

Now, we estimate the radial electric field by a simple analytical model. The collision term $C(f, f)$ is neglected since the radial electric field is built up in a shorter time than the collision time. We take the time derivative of Eq.(3) to yield

$$\left(1+\frac{c^2}{v_A^2}\right)\frac{\partial^2 E_r}{\partial t^2} = -4\pi e\left\langle \int d^3v \frac{\partial f}{\partial t} v_{dr} \right\rangle \quad (5)$$

where we asuumed that $1 + c^2/v_A^2$ = const. The drift velocity $v_{dr} = -(v_{||}^2 + v^2)/(2\Omega_i R)\sin\theta$ is assumed to be constant in Eq.(4), because f oscillates with the GAM frequency and its time change is much faster than that of v_{dr}. The time change in f is approximated by the variation in f_{SM}. Then, Eq.(5) can be written as

$$\frac{\partial^2 E_r}{\partial t^2} + \frac{e}{T}C_1 E_r = C_2 \quad (6)$$

The factors C_1 and C_2 can be easily calculated assuming $\nabla T = 0$. In the limit of $t = \infty$ (after the GAM damps out), $E_r = (T/e)(C_2/C_1)$ and we obtain

$$E_r = \frac{T}{e}\left[\frac{\nabla n}{n} + 2\left(\frac{I_1}{I_0} - \bar{u}_{||}\frac{d\bar{u}_{||}}{dr}\right)\right] + \frac{T}{e}\alpha\frac{I_2}{I_0}\bar{u}_{||} \quad (7)$$

where $\alpha = 2r/(qR\rho)$, $\bar{u}_{||} = u_{||}/v_{th}$, and I_0, I_1 and I_2 are integrals containing Gauss function to yield $I_1/I_0 \simeq 3.3\bar{u}_{||}$ and $I_2/I_0 \simeq 0.43$.

Calculation parameters are; B_0 = 3 T, R_0 = 3 m, a_p = 0.5 m, $n_0 = 2.5\times10^{19}$ m^{-3}, and T_i = 1.0 keV. The density profile is smooth and the temperature is constant. The assumed profile of parallel flow of plasma ions is

$$\bar{u}_{||}(r) = u_{||0}\exp\left[-2(x-x_0)^2\right] \quad (8)$$

In Fig. 1, the calculation results are shown. Three cases of (1) $u_{||0} = 0$, (2) $u_{||0} = +0.1$, and (3) $u_{||0} = -0.1$ are illustrated. Simulation results (solid lines) and analytical estimations (dotted lines) are well in agreement. The agreement can be seen for the case of the plasma with steep pressure gradient. The comparison with an experimental measurement of E_r in a rotating plasma is under investigation.

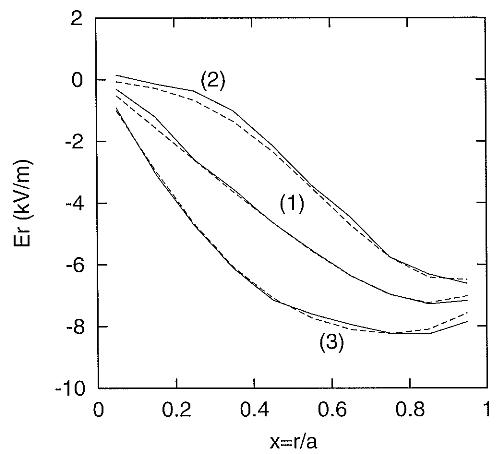

Fig. 1 E_r profiles in a rotating plasma.

[1] S. Satake, H. Sugama, M. Okamoto and M. Wakatani, J. Plasma Fus. Res. **77**, 573 (2001).

[2] Z. Lin, M. W. Tang and W. W. Lee, Phys. Plasmas **4**, 1707 (1997).

[3] W. X. Wang, N. Nakajima, M. Okamoto and S. Murakami, Plasma Phys. Control. Fusion **41**, 1091 (1999).

[4] W. X. Wang, F. L. Hinton and S. K. Wong, Phys. Rev. Let. **87** (5), 055002-1 (2001).

[5] S. Brunner, E. Valeo and J. A. Krommes, Phys. Plasmas **6** (12), 4504 (1999).

[6] N. Winsor, J. L. Jhonson and J. M. Dawson, Phys. Fluid **11** (11), 2448 (1968).