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# Localized Stability Criterion and Double Adiabatic Theory

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The stability criterion for the localized interchange mode is reexamined on the basis of the magnetohydrodynamics with double adiabatic laws. In plasma with double adiabaticity, the localized interchange mode is stabilized significantly due to coupling with the sound mode. This stabilization effect may explain the observed stability of LHD plasma beyond the predicted stability boundary.

**Keywords:** LHD, MHD stability, mercier criterion, double adiabatic theory, suydam condition

Magnetohydrodynamic (MHD) stability is regarded as one of the most important issues in magnetic confinement of fusion plasma. In theoretical investigation of the MHD stability of helical torus plasma, the Mercier stability criterion [1] for the localized mode is used as a convenient measure of the instability.

In the LHD experiment [2], plasma confinement improves due to inward shifts of the plasma, as is expected theoretically; and much effort has been devoted to pursuing further improvements by achieving larger inward plasma shifts. Current theory predicts strong instability in cases of inward plasma shifts of over 15 - 20 cm [3]. Experimentally, however, inward shifts of even 30 - 40 cm can be made without creating severe instability [4].

It is of great importance to understand the origins of this conflict between theory and experiment. The predicted instability is so strong that it is unplausible to explain by changes in pressure profiles or toroidal current profiles. We propose here a plasma model based on double adiabaticity as a possible explanation for stability of LHD plasma beyond the Mercier criterion.

The collisionless plasma placed in the strong magnetic field obeys the double adiabatic laws [5]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_{//}B^2}{\rho^3}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{\rho B}\right) = 0. \tag{1}$$

Here, $p_{//}$ and $p_\perp$ stand for the pressure parallel and perpendicular to the magnetic field, and $\rho$ is the mass density. Conventional MHD theory assumes the single adiabatic law, which implies frequent collisions mixing the freedoms of the particle motion in different directions. When the plasma temperature rises, the plasma becomes more collision-free, and the parallel motion along the magnetic field lines and the gyration around the magnetic field become more and more independent of each other; it is possible, however, that descriptions of motion based on double adiabatic laws may be more appropriate for such plasmas. The double adiabaticity is treated primarily in the mirror plasma as well as in toroidal plasma with anisotropic plasma pressure. Here, we assume that the equilibrium plasma pressure is isotropic $p_{//} = p_\perp = p(\psi)$; only the perturbed pressure may be anisotropic. In this case, the potential energy in the double adiabatic MHD can be written as [6]

$$\delta W = \int_{\Omega_p} \{W_B + W_p\}\,\mathrm{d}\tau, \tag{2}$$

with

$$\begin{aligned} \overline{W}_B &= |\boldsymbol{Q}|^2 + \boldsymbol{J}\times\boldsymbol{\xi}\cdot\boldsymbol{Q} + (\boldsymbol{\xi}\cdot\nabla p)\nabla\cdot\boldsymbol{\xi}, \\ \overline{W}_p &= \tfrac{5}{3}p(\nabla\cdot\boldsymbol{\xi})^2 + \tfrac{1}{3}p[\nabla\cdot\boldsymbol{\xi} - 3q]^2, \end{aligned} \tag{3}$$

where $\boldsymbol{\xi}$ is the plasma displacement,

$$\boldsymbol{Q} \equiv \nabla\times(\boldsymbol{\xi}\times\boldsymbol{B}), \quad q \equiv \boldsymbol{b}\cdot(\boldsymbol{b}\cdot\nabla)\boldsymbol{\xi}, \tag{4}$$

and $\boldsymbol{b}$ is the unit vector along the magnetic field. In conventional MHD with single adiabaticity, the marginal stability is characterized as $\nabla\cdot\boldsymbol{\xi} = 0$; the instability decouples with the sound wave. This is not true in the case of double adiabaticity, and the coupling with the sound wave may significantly stabilize the interchange mode.

When the cylindrical plasma is considered, the stability condition of the localized mode (Suydam condition [7]) is transformed into the form

$$\frac{1}{4}\left(\frac{r\mu'}{\mu}\right)^2 + \frac{2r}{B_z^2}\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\frac{5}{3}p}{B^2+\frac{5}{3}p} > 0 \tag{5}$$

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in the MHD with double adiabaticity. The last term is manifested by the double adiabaticity, which results in stability beyond the Suydam limit.

We shall examine the localized mode in the toroidal geometry. In the vicinity of the rational surface with $\iota_0 \equiv \iota(\psi_r) = n_0/m_0$, the stretched coordinates are introduced.

$$x = \varepsilon^{-1}(\psi - \psi_r), \quad y = \theta - \theta_0 - \iota_0\phi, \quad \zeta = \phi. \quad (6)$$

By expanding the plasma displacement

$$\boldsymbol{\xi} = \xi_\psi \frac{\nabla\psi}{|\nabla\psi|^2} + \xi_s \frac{\boldsymbol{B}\times\nabla\psi}{B^2} + \xi_b \boldsymbol{B} \quad (7)$$

in the form

$$\begin{aligned} \xi_\psi &= \xi_\psi^{(0)}(x,y,\zeta) + \varepsilon\xi_\psi^{(1)}(x,y,\zeta) + \cdots, \\ \xi_s &= \varepsilon^{-1}\left\{\xi_s^{(0)}(x,y,\zeta) + \varepsilon\xi_s^{(1)}(x,y,\zeta) + \cdots\right\}, \quad (8) \\ \xi_b &= \varepsilon^{-1}\left\{\xi_b^{(0)}(x,y,\zeta) + \varepsilon\xi_b^{(1)}(x,y,\zeta) + \cdots\right\}, \end{aligned}$$

the plasma potential energy is minimized step by step. The plasma pressure is assumed as small as $p \sim O(\varepsilon^2)$. The lowest order potential energy can be minimized for the displacement such as

$$\xi_\psi^{(0)} = \frac{\partial\Phi}{\partial y}, \quad \xi_s^{(0)} = -\frac{\partial\Phi}{\partial x}, \quad (9)$$

where $\Phi$ is the electrostatic potential independent of $\zeta$. The minimization with respect to $\xi_\psi^{(1)}$ and $\xi_s^{(1)}$ can be carried out similar to the single adiabatic case. The minimization with respect to $\xi_b^{(0)}$ creates difficulty, as the ripple of the magnetic field strength along the magnetic line of force causes coupling with the sound wave, making it necessary to solve a second order differential equation. If we assume a small magnetic ripple, i.e. $h \sim \delta \ll 1$ where $h \equiv \boldsymbol{b}\cdot\nabla B$, the second order differential equation can be solved by approximation, and the expression for the plasma potential energy density is obtained as

$$\overline{W}^{(0)} \propto s^2x^2\left|\frac{\partial\xi_\psi^{(0)}}{\partial x}\right|^2 - D\left|\xi_\psi^{(0)}\right|^2 + C_{DA}\left|\xi_s^{(0)}\right|^2, \quad (10)$$

where $s \equiv \iota_0'$ is the magnetic shear, $D$ is the coefficient appearing in the Mercier criterion ($\frac{1}{4}s^2 - D \geq 0$), and

$$C_{DA} = \frac{5}{3}p\left\langle\frac{B^2}{|\nabla\psi|^2}\right\rangle\left\{\left\langle\kappa_s{}^2\right\rangle - \frac{\langle\kappa_s h\rangle^2}{\langle h^2\rangle}\right\}. \quad (11)$$

Here $\kappa_s$ is the geodesic curvature

$$\kappa_s = \frac{\boldsymbol{B}\times\nabla\psi}{2B^4}\cdot\nabla B^2, \quad (12)$$

with the bracket indicating the average along the closed magnetic line of force:

$$\langle A\rangle = \oint_c \frac{A\,\mathrm{d}\ell}{B} \Big/ \oint_c \frac{\mathrm{d}\ell}{B}. \quad (13)$$

Because the expression (11) is always positive, the last term in eq. (10), which is not in the single adiabaticity MHD, act as a stabilizer. Due to this stabilization term, the size of localization around the rational surface becomes bounded.

By using the potential energy and kinetic energy terms, we can construct the eigenvalue problem. If we denote the real frequency for the single adiabaticity as $\omega_{MHD}$ and that for the double adiabaticity as $\omega_{DA}$, we can obtain the relation

$$\omega_{DA}{}^2 = \frac{\omega_{MHD}{}^2\left\langle\frac{\rho|\nabla\psi|^2}{B^2}\right\rangle + C_{DA}}{\left\langle\frac{\rho|\nabla\psi|^2}{B^2}\right\rangle + \frac{\langle\rho B^2\rangle\langle h\kappa_s\rangle^2}{\langle h^2\rangle^2}}. \quad (14)$$

In conclusion, in the highly collisionless plasma, the MHD with double adiabatic laws may be more plausible than MHD with single adiabatic law; in the MHD with double adiabatic laws the interchange instability localized around the rational surface does not occur because of coupling with the slow sound mode. The lack of instability in LHD configuration in inwardly shifted configuration might be explained by this effect.


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