Effects of Alfvén Ion Cyclotron Modes on Tandem Mirror Energy Transport

HOJO Hitoshi, NAKAMURA Motoyuki, TANAKA Satoru, ICHIMURA Makoto and MASE Atsushi
Plasma Research Center, University of Tsukuba, Tsukuba 305-8577, Japan

(Received 12 January 1999 / Accepted 15 July 1999)

Abstract
An energy transport model taking into account the effects of Alfvén ion cyclotron fluctuations is presented to explain the hot-ion-mode experiments of tandem mirror GAMMA 10. The energy transport equations for the electron and ion including the ion temperature relaxation due to the quasilinear effects of unstable Alfvén ion cyclotron modes are derived based on a point model and are solved numerically for the steady state. It is shown that the numerical results obtained from the present model can well explain the experimental results in the hot-ion-mode of GAMMA 10.

Keywords:
tandem mirror, GAMMA 10, hot ion mode, Alfvén ion cyclotron mode, transport study, energy transport, quasilinear temperature relaxation

1. Introduction
In the hot-ion-mode experiments of tandem mirror GAMMA 10, strongly anisotropic hot ions in the central cell are produced by the Ion Cyclotron Range of Frequencies (ICRF) heating [1,2]. Namely, the perpendicular temperature $T_{ii}$ of the ion is 3–10 keV, and the parallel temperature $T_{ni}$ is several hundreds eV. Then, the temperature anisotropy $T_{ii}/T_{ni}$ of the ion becomes to be about 10 or more. In GAMMA 10, Alfvén Ion Cyclotron (AIC) modes have been observed to be excited due to this strong temperature anisotropy of the ion [3,4]. On the other hand, the electron temperature $T_{e}$ in the central cell is typically 0.1–0.2 keV, and the typical value of $T_{ii}/T_{e}$ is roughly larger than 3 in the absence of Electron Cyclotron Resonance Heating (ECRH).

Our aim in this paper is to make clear what kinds of transport processes dominate to determine the ion and electron temperatures in the hot-ion-mode experiments of GAMMA 10. We now study energy transport processes in the hot-ion-mode experiments of GAMMA 10 and try to reproduce theoretically the experimental results. We here present a model of zero-dimensional energy transport equations for the electron and ion based on a point model. The model includes a quasilinear relaxation process for the ion temperature due to the unstable AIC modes, in addition to standard classical processes such as collisional temperature relaxation, collisional axial loss to the end and charge exchange loss. We here assume the existence of an additional energy loss channel besides collisional axial loss to the end and the electron drag in the energy loss processes for the cold ion with $T_{ni}$. This additional energy loss for the cold ion with $T_{ni}$ is of importance, since it forces to obstruct the increase in $T_{ii}$ which is produced by the temperature relaxation from $T_{ii}$ due to collisions or AIC modes. As the candidate for this additional energy-loss mechanism, radial transport of the passing cold ion in

corresponding author's e-mail: hojo@prc.tsukuba.ac.jp
*Present Address: Advanced Science and Technology Center for Cooperative Research, Kyushu University, Kasuga 816-8580, Japan
†この論文は第15回年会にて招待講演として発表されたものを論文化したものです。
the transition region [5], or anomalous axial loss to the end driven by the unstable AIC modes [6] might be considered. We solve numerically the energy transport equations for the electron and ion in the steady state, and then show that the present model can well reproduce the experimental results in the hot ion modes of GAMMA 10. We then confirm that the both effects (the quasilinear temperature relaxation from $T_{ii}$ to $T_{ui}$ due to the unstable AIC modes, and the additional energy loss for the cold ion with $T_{ni}$ such as radial loss or anomalous axial loss due to unstable waves besides collisional axial loss to the end) are of very importance in the explanation of the hot-ion-mode experiments of GAMMA 10.

In the following section, we briefly review the AIC mode instability and the resultant quasilinear effects on temperature relaxation. In Sec. 3, we derive the zero-dimensional energy transport equations for the electron and ion including the quasilinear effects of unstable AIC modes, which are solved numerically for the steady state in Sec. 4. In Sec. 5, we summarize the results obtained in this paper and also briefly mention critical issues omitted in this paper.

2. Alfvén Ion Cyclotron Modes

In this section, we discuss AIC modes, which are observed to be excited in the hot ion mode experiment of GAMMA 10. The AIC mode is left-hand circularly-polarized mode and is destabilized by the temperature anisotropy of the ion for a finite beta plasma [7,8]. The linear dispersion equation for the AIC modes is given by

$$\epsilon_L(\omega, k, T_{ii}, T_{ui}) = 1 - \frac{ck}{\omega} \left[ \frac{\omega_{ci}^2}{\omega} - \frac{\omega_{ci}}{\omega + \omega_{ci}} \right]$$

$$+ \frac{\omega_{ci}^2}{\omega^2} \left[ \frac{\omega - \omega_{ci}}{k v_{li}} \right] Z \left( \frac{\omega - \omega_{ci}}{k v_{li}} \right)$$

$$+ \frac{1}{2} \left( 1 - \frac{T_{li}}{T_{ui}} \right) Z' \left( \frac{\omega - \omega_{ci}}{k v_{li}} \right) = 0 ,$$

where $\omega$ the wave frequency, $k$ the parallel wave number, $\omega_{ci} (= \sqrt{2}n/m_i e_c)$ is the plasma frequency, $\omega_{ci} (= q_i B_0/m_i)$ the cyclotron frequency, $v_{th} (= \sqrt{2}T_{li}/m_i)$ the ion thermal speed, $c$ the speed of light, $q_i$ the charge ($q_i = e, q_i = -e$), $m$ the mass, $n$ the plasma density, $B_0$ the external magnetic field, $Z(x)$ the plasma dispersion function defined by

$$Z(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt$$

and $Z'(x) = dZ/dx$. In the derivation of eq.(1), it is assumed that the electron is cold and the ion has a bi-Maxwellian distribution with $T_{li}$ and $T_{ui}$. This assumption is satisfied as we have $T_{li} \gg T_{ui} > T_i$ in the hot-ion-mode experiments of GAMMA 10.

When we here introduce the ion distribution function $f_i(v, t)$, the perpendicular and parallel temperatures of the ion, $T_{ii}$ and $T_{ui}$ are defined as

$$n T_{ii} = \frac{1}{2} m_i v_{li}^2 f_i(v, t) dv ,$$

$$\frac{1}{2} n T_{ui} = \frac{1}{2} m_i v_{ui}^2 f_i(v, t) dv .$$

The time evolution of the ion temperature due to the quasilinear effects of unstable AIC modes is then described by the following equations:

$$\frac{d}{dt} (n T_{ii}) = -2 \int 2\gamma(k, t) E(k, t) dk ,$$

$$\frac{d}{dt} \left( \frac{1}{2} n T_{ui} \right) = \int 2\gamma(k, t) E(k, t) dk ,$$

where $E(k, t) = B_k/2\mu_0$, $B_k$ being the Fourier component of the magnetic fluctuation, is the spectral wave-energy density of the AIC mode, and $\gamma(k, t)$ is the growth rate of the AIC mode obtained from eq.(1). In the derivation of eq.(3), for simplicity, we assumed $(\omega_{ci}ck)^2 \ll 1$ and $(\omega/c)^2 \ll 1$, which are satisfied for the AIC modes observed in GAMMA 10. We also assume that the electron has no interactions with the AIC mode as the electron temperature is low in the hot-ion-mode experiments, and that the spectral wave-energy density $E(k, t)$ obeys the following wave kinetic equation:

$$\frac{d}{dt} E(k, t) = 2\gamma(k, t) E - \alpha(k) E^2 ,$$

where we introduce a nonlinear damping term $-\alpha E^2$ in eq.(4) to get the saturation of the unstable AIC modes. The temperature relaxation of the ion due to the quasilinear effects of the unstable AIC modes being
Effects of Alfvén Ion Cyclotron Modes on Tandem Mirror Energy Transport

In Fig. 1, we show the eigenfrequencies of the AIC modes calculated from eq.(1), where $\beta (= 2\mu_0 n T_{ii} / B_0^2) = 0.01$, $T_{ii}/T_{ni} = 10$ (solid line) and 14 (dashed line) are used. We see that the growth rate becomes larger for the stronger anisotropy of the ion temperature. We also see that the growth rate becomes negative for a large value of $ck/\omega_{ci}$ due to the cyclotron resonance damping, and rapidly decays to zero for $ck/\omega_{ci} \leq 2$. The detailed linear analysis on the AIC-mode instability has been discussed in ref.7.

3. Energy Transport Model

In this section, we construct a model for energy transport in a hot-ion-mode plasma of the tandem mirror GAMMA 10 based on a point model. We here assume that the plasma is sustained by ICRF only, and that there is no ECRH for ion-confining potential formation in the barrier cell. We consider a model of power flow chart shown in Fig. 2, where the temperature relaxation of the ion due to the quasilinear effects of the unstable AIC modes discussed in the previous section are included. Then, we assume the following energy transport equations for the electron and ion in the hot-ion-mode plasma of GAMMA 10 as

$$\frac{d}{dt}(nT_{li}) = \frac{\nu}{\tau_{cx}} - n v(T_{li} - T_i) - 2 \int_{-}^{+} dk 2\gamma E - \frac{n(T_{li} - T_e)}{\tau_{el}},$$

$$\frac{d}{dt} \left( \frac{nT_{ii}}{2} \right) = n v(T_{li} - T_{ii}) + \int_{-}^{+} dk 2\gamma E - \frac{nT_{ii}}{2\tau_{ii}} - \frac{nT_{ii}}{2\tau_{ad}} - \frac{n(T_{ii} - T_e)}{2\tau_{ei}},$$

$$\frac{d}{dt}(nT_e) = \frac{n}{\tau_{el}} \left( \frac{2T_{li} + T_{ii}}{3} - T_e \right) - \frac{nT_e}{\tau_{el}},$$

where $P_{RF}$ is the ICRF power, $\tau_{cx}$ the charge exchange time, $\tau_{el}$ the electron drag time, $\tau_{ii}$ the collisional axial (energy) confinement time for the cold ion with $T_{ii}$, $\tau_{ad}$ the confinement time for an additional energy loss of the cold ion such as radial loss or anomalous axial loss due to the unstable AIC modes, $\tau_{ei}$ the collisional axial (energy) confinement time for the electron [10], and $v$ the temperature relaxation rate due to collisions [11]. The characteristic times (in second) are given by

![Fig. 1: Eigenfrequencies of the AIC mode calculated from eq.(1), where $T_{ii}/T_{ni} = 10$ (solid line), 14 (dashed line) and $\beta = 0.01$.](image)

![Fig. 2: Power flow chart in the present model to explain the hot-ion-mode experiment of GAMMA 10.](image)
\[ \tau_{\alpha} [s] = 0.32 T_e^{3/2} , \]
\[ \tau_{\parallel} [s] = 0.03 T_{\parallel}^{3/2} . \]
\[ \tau_{ep} [s] = 5.4 \times 10^{-4} T_e^{3/2} \frac{\phi_e}{T_e} \exp \left( \frac{\phi_e}{T_e} \right) , \tag{8} \]
\[ \tau_{ep} = \frac{3}{2} \frac{\tau_{ep}}{\phi_e/T_e + 1} , \]
\[ \nu [s^{-1}] = \frac{90}{T_{\parallel}^{3/2}} \frac{1}{A^2} \left[ -3 + (A + 3) \frac{\arctan \sqrt{A}}{\sqrt{A}} \right] , \]
\[ A = \frac{T_{\parallel}}{T_e} - 1 , \]

where temperatures and \( \phi_e \) (being the ambipolar potential for electron confinement) are shown in keV and \( n = 2 \times 10^{12} \text{ cm}^{-3} \) for the density is assumed as its typical value. Here, the axial energy confinement time \( \tau_{ed} \) is related to the axial particle confinement time \( \tau_{ep} \) through the above relation. We here note that the effect of magnetic mirror for the GAMMA 10 magnetic configuration is taken into account for the numerical coefficients in the characteristic times given by eq.(8).

4. Steady-State Power Flow

In this section, we discuss the steady-state power flow in the GAMMA 10 plasma based on the energy transport model obtained in the previous section. We first consider the ambipolar potential \( \phi_e \) for electron confinement. The ambipolar potential is determined from the ambipolar condition that the particle losses of the electron just balance to those of the ion, and is given by \( n/\tau_{ep} = n/\tau_{ed} + n/\tau_{ed} \). This ambipolar condition is reduced to the following equation:

\[ \frac{C_i}{C_e} \left( \frac{T_{\parallel}}{T_e} \right)^{3/2} = \left( 1 + \frac{T_{\parallel}}{T_{ed}} \right) \frac{\phi_e}{T_e} \exp \left( \frac{\phi_e}{T_e} \right) , \tag{9} \]

with \( C_i = 0.03 \) and \( C_e = 5.4 \times 10^{-4} \). The potential \( \phi_e \) is then expressed as a function of \( T_{\parallel} \) and \( T_e \).

We next consider the spectral wave-energy density \( E(k) \) of the AIC modes to estimate the integral of \( E(k) \) in eqs.(5) and (6). For simplicity, we here assume that the spectral wave-energy density \( E(k) \) is proportional to the linear growth rate \( \gamma(k) \). Then, we can obtain \( E(k) = 2 \gamma(k)/\alpha(k) \) from eq.(4) in the steady state, and also assume that the nonlinear coefficient \( \alpha \) is constant. In this case, the integral of the spectral wave-energy density in eqs.(5) and (6) is given by

\[ \int_{-\infty}^{\infty} \frac{dk \gamma^2(k)}{\gamma(k)} = W \int_{-\infty}^{\infty} dk \gamma(k) , \]

\[ W = \int_{-\infty}^{\infty} dk E(k) , \]

where \( W \) is the total wave energy density of the AIC modes. Then, when \( \tau_{ex} \), \( \tau_{ed} \) and \( W \) are given, we can determine \( T_{\parallel}, T_e, T_i, \) and \( \phi_i \) by solving eqs.(5)-(7) and (9) simultaneously. Hereafter, we assume \( \tau_{ex} = 10 \text{ ms} \) and \( n = 2 \times 10^{12} \text{ cm}^{-3} \) as the typical values of \( \tau_{ex} \) and \( n \) in the hot-ion-mode experiments. We now show the numerical results.

In Fig.3, we show \( T_{\parallel} \) (closed circle), \( T_e \) (closed square), \( T_{\parallel}/T_i \) (open circle), and \( T_{\parallel}/T_e \) (open square) as a function of \( T_i \) in the case of \( T_{ex} = 4 \text{ ms} \) and \( W/n = 10^{-4} \text{ keV} \). This value in \( W \) for the AIC modes roughly corresponds to the magnetic fluctuations of about 0.2 gauss. This magnitude of the magnetic fluctuation is comparable to that estimated experimentally from the measurements of the AIC modes by reflectometers [12,13]. The parallel ion and electron temperatures

![Fig. 3](https://via.placeholder.com/150)

Fig. 3 \( T_{\parallel}, T_e, T_{\parallel}/T_i \) and \( T_{\parallel}/T_e \) as a function of \( T_i \) for \( \tau_{ex} = 10 \text{ ms}, \tau_{ed} = 4 \text{ ms} \) and \( W/n = 10^{-4} \text{ keV} \).
increase with the increase of the perpendicular ion temperature. In this case, we obtain the temperature ratios of $T_{\parallel i} / T_{\perp i} \approx 9$ and $T_{\parallel i} / T_e \approx 3-4$ for $T_{\parallel i} = 3-6$ keV. These temperature ratios are close to the experimental results on the hot-ion-modes of GAMMA 10.

On the other hand, we show $T_{\parallel i}$, $T_e$, $T_{\parallel i} / T_{\perp i}$, and $T_{\parallel i} / T_e$ as a function of $T_{\perp i}$ in the case of $\tau_{ad} = \infty$ and $W = 0$ in Fig.4, where the used marks are the same as those in Fig.3, in order to compare with Fig.3. In this case, we obtain the temperature ratios of $T_{\parallel i} / T_{\perp i} = 6$ and $T_{\parallel i} / T_e = 2$ for $T_{\perp i} = 3-6$ keV. These temperature ratios greatly differs from the experimental results on the hot-ion-mode operation of GAMMA 10. In this case, we see that the electron and parallel ion temperatures become larger than those values in Fig.3 for a given $T_{\perp i}$. The reason is as follows: As the confinement time for the cold ion component becomes much longer than that in the case of Fig.3 due to the absence ($\tau_{ad} = \infty$) of the additional energy-loss channel, the parallel ion and electron temperatures can become larger than those in the case of Fig.3 by the collisional energy relaxation alone from $T_{\perp i}$, even if the energy relaxation due to the quasilinear effects of the unstable AIC modes vanishes.

In Fig.5, we show $T_{\parallel i}$ (circles), $T_e$ (squares) as a function of $T_{\perp i}$ in the case of $\tau_{ad} = \infty$, $W = 0$ (closed circle or square) and $W/n = 10^{-4}$ keV (open circle or square). Here, the increments in $T_{\parallel i}$ and $T_e$ due to the finite $W$ are obviously considered to be brought by the quasilinear temperature relaxation due to the AIC modes. We see that the temperature relaxation due to the AIC modes is more effective in $T_{\parallel i}$ rather than in $T_e$. We also see from Fig.5 that if $\tau_{ad} = \infty$, that is, the energy losses of the ion and electron are limited to only the charge exchange loss and the classical collisional axial losses to the end, it is expected that we obtain the parallel ion and electron temperatures higher than those obtained in the hot-ion-mode experiments of GAMMA 10.

Finally, we argue the numerical results for the case of $\tau_{ad} = 4$ ms and $W = 0$. In this case, it is found that there is no solution of $T_{\perp i}$ in the region of $10$ eV $< T_{\perp i} < 3$ keV (and $3$ keV $< T_{\parallel i} < 6$ keV) which can satisfy eqs.(5) to (7) in the steady state. This is because the energy balance of eq.(6) cannot be satisfied in the steady state as the total energy loss becomes always larger than the energy input from collisional temperature relaxation. Therefore, the experimental situation such as $\tau_{ad} = 4$ ms and $W = 0$ is never realized in the hot-ion-mode experiments of GAMMA 10.

### 5. Summary

In this section, we summarize the results of energy transport study on the hot-ion-mode experiments of GAMMA 10 obtained in this paper. We have presented an energy transport model taking into account the quasilinear effects of the unstable AIC modes observed in GAMMA 10 experiments, and have shown numerically that the present model of energy transport can well explain the experimental results in the hot-ion-
mode operation of GAMMA 10. The important elements necessary to reproduce the experimental results are found to be (1) the quasilinear temperature relaxation effects due to the AIC modes and (2) the existence of energy loss for the cold ion with $T_i$, such as radial loss or anomalous axial loss due to unstable waves in addition to the collisional axial loss to the end.

The following issues are listed as the remaining problems on energy transport in GAMMA 10: 1. To make clear whether there exist interactions of the AIC modes with the electron or not. 2. To make clear whether there exists an additional energy loss channel for the electron besides the classical collisional loss to the end or not. 3. To taking into account the effects of ECRH for ion-confining potential formation in the barrier cell. Works on these issues will be reported elsewhere in future.

**References**


