研究論文

Density Fluctuations of Electromagnetic Plasma Waves in Ion Cyclotron Range of Frequencies

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(Received May 31, 1993)

Abstract

Relationship between the density and magnetic field fluctuations of electromagnetic plasma waves in an infinitely uniform magnetized plasma is studied based on MHD, two-fluid and kinetic linear theories. In the MHD theory, density fluctuation is related to parallel magnetic field fluctuation. The density fluctuations of the shear and compressional Alfvén waves are discussed. The two-fluid theory includes parallel electric field fluctuations. The breakdown due to the lack of wave-particle resonances is discussed. In the kinetic theory, the relationship including wave-particle resonances is obtained from the Vlasov equation. The kinetic expression is compared with experimental results for the Alfvén ion cyclotron waves observed in GAMMA 10 by means of the reflectometer measurement.

Keywords:

density fluctuation, magnetic field fluctuation, electromagnetic plasma wave, Alfvén wave, Alfvén ion cyclotron wave, wave-particle resonance, reflectometer measurement, GAMMA 10,

1. Introduction

It is well known that electrostatic plasma waves are originated by density fluctuations. On the other hand, although so-called electromagnetic plasma waves are casually thought to be free of density fluctuations, it is generally unclear whether electromagnetic plasma waves cause density fluctuations or not. We need analyses to find out the answer.

The aim of this paper is to discuss the relationship between the density and magnetic field fluctuations associated with electromagnetic plasma waves. We here consider electromagnetic plasma waves with frequencies in the ion cyclotron range of frequencies (ICRF) in an infinitely uniform magnetized plasma. In section 2, the magnetohydrodynamic (MHD) theory approach is presented. We analyze the density fluctuations associated with the shear and compressional Alfvén waves in a cold plasma and obtain the relationship between the density and magnetic field fluctuations. In section 3, we discuss the relationship based on the two-fluid theory in a hot plasma. We here include the contribution of electric field fluctuations, which is neglected in the MHD theory. We also mention the breakdown of the two-fluid theory due to the lack of wave-particle resonance effects. In section 4, we deal with the kinetic theory. We here derive the relationship between the density and magnetic field fluctuation from the linearized Vlasov equation, where the wave-particle resonance effects are taken into account. We see that the results of the kinetic theory cover those of the two-fluid theory. The wave-particle resonances describe the Landau damping and transit time damping by the electron. In section 5, concerning with the ratio of the density to magnetic field fluctuations. We compare the expression by the kinetic theory with the recent experimental result of Alfvén ion cyclotron (AIC) fluctuations observed in GAMMA 10 by means of measurement using both O-mode and X-mode reflectometers. Finally, in section 6, we summarize the results obtained in this paper.

2. MHD Theory

In this section, we discuss the relationship between the density and magnetic field fluctuations associated with electromagnetic plasma waves in the ideal MHD theory, which is valid for very low-frequency fluctuations. We here assume perturbed quantities with the plane-wave form given by $\exp[i(\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp + k_\parallel z - \omega t)]$ for simplicity, where ω is the wave frequency, $\boldsymbol{k}_\perp$ and $k_\parallel (= \hat{z} \cdot \boldsymbol{k})$ the components of the wave number $\boldsymbol{k}$ perpendicular and parallel to an unperturbed magnetic field $\boldsymbol{B}_0 = B_0 \hat{z}$, respectively. The vector $\hat{z}$ is the unit vector in the z-direction.

We here assume an infinitely uniform cold plasma. We also assume that the unperturbed magnetic field $\boldsymbol{B}_0$ is uniform and the wave frequency ω is much smaller than the ion cyclotron frequency ω_{ci} $(= eB_0/M_i)$, where e is the ion charge and M_i the ion mass. Then, small perturbations of the plasma are described by the linearized MHD equations given by

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\rho_0 \tilde{\boldsymbol{v}}) = 0, \tag{1}$$

$$\rho_0 \frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = \tilde{\boldsymbol{j}} \times \boldsymbol{B}_0, \tag{2}$$

$$\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{v}} \times \boldsymbol{B}_0 = 0, \tag{3}$$

$$\nabla \times \tilde{\boldsymbol{E}} = -\frac{\partial \tilde{\boldsymbol{B}}}{\partial t}, \tag{4}$$

$$\nabla \times \tilde{\boldsymbol{B}} = \mu_0 \tilde{\boldsymbol{j}} \tag{5}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ the magnetic field, $\boldsymbol{v}$ is the flow velocity, $\boldsymbol{j}$ the current density and ρ the mass density and μ_0 the permeability of vacuum. Here, the suffix 0 denotes an equilibrium quantity and the suffix ~ denotes a perturbed quantity. We obtain, from eqs. (1) through (3),

$$\frac{\partial}{\partial t} \frac{\tilde{\rho}}{\rho_0} + \nabla_\perp \cdot \tilde{\boldsymbol{v}}_\perp + \frac{\partial}{\partial z} \tilde{v}_\parallel = 0, \tag{6}$$

$$\tilde{\boldsymbol{v}}_\perp = \frac{\tilde{\boldsymbol{E}} \times \boldsymbol{B}_0}{B_0^2}, \tag{7}$$

$$\tilde{v}_\parallel = 0. \tag{8}$$

Substituting eqs. (7) and (8) into eq. (6), we obtain

$$\frac{\partial}{\partial t} \frac{\tilde{\rho}}{\rho_0} + \frac{1}{B_0} \hat{z} \cdot (\nabla \times \tilde{\boldsymbol{E}}) = 0. \tag{9}$$

Since $\rho \cong M_i n$, where n is the plasma density, the substitution of eq. (4) into eq. (9) yields the relation between the density and magnetic field fluctuations as

$$\frac{\tilde{n}}{n_0} = \frac{\tilde{B}_\parallel}{B_0}, \tag{10}$$

where $\tilde{B}_\parallel (= \hat{z} \cdot \tilde{\boldsymbol{B}})$ is the parallel component of the magnetic field fluctuation. In the ideal MHD theory for a uniform cold plasma, we see that the density fluctuation is related to the parallel component of the magnetic field fluctuation. The perpendicular component $\tilde{\boldsymbol{B}}_\perp$ of the magnetic field fluctuation does not cause the density fluctuation.

We also obtain, from eqs. (1) through (5),

$$\tilde{E}_\parallel = 0, \tag{11}$$

and two eigenmodes as follows:

$$\omega = V_A k_\parallel \quad \text{with} \quad \tilde{B}_\parallel = 0, \tag{12}$$

$$\omega = V_A (k_\parallel^2 + k_\perp^2)^{1/2} \quad \text{with} \quad \tilde{B}_\parallel \neq 0, \tag{13}$$

where $V_A (= B_0/\sqrt{\mu_0 \rho_0})$ is the Alfvén velocity. For the former mode (12), the density fluctuation does not arise because of $\tilde{B}_\parallel = 0$. Therefore, it is called as the shear (or torsional) Alfvén wave. On the other hand, the latter mode (13) induces the density fluctuation and therefore is called as the compressional Alfvén wave.

We next consider the Alfvén waves near the ion cyclotron frequency. When we take into account the Hall effect, eq. (3) is replaced by [1]

$$\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{v}} \times \boldsymbol{B}_0 = \frac{1}{en_0} \tilde{\boldsymbol{j}} \times \boldsymbol{B}_0. \tag{14}$$

In this case, we obtain again eq. (11) and two eigenmodes, i.e., the shear and compressional Alfvén waves. We note that in this case the parallel magnetic field fluctuation $\tilde{B}_\parallel$ for the shear Alfvén wave does not vanish. That is, this means that the shear Alfvén wave as well as the compressional Alfvén wave also induces the density fluctuation when the wave frequency approaches near the ion cyclotron frequency. For both the shear and compressional Alfvén waves, the parallel magnetic field fluctuation $\tilde{B}_{\parallel}$ is expressed by the Bessel function $J_m(k_\perp r)$ in an infinite cylindrical plasma, where m denotes the azimuthal mode number and the perpendicular wave number $k_\perp$ is given by

$$k_\perp^2 = A - k_\parallel^2 - \left(\frac{\omega}{\omega_{ci}}\right)^2 \frac{A^2}{A - k_\parallel^2}, \tag{15}$$

$$A = \frac{\omega^2}{V_A^2(1-\omega^2/\omega_{ci}^2)}. \tag{16}$$

From eqs. (15) and (16), we can obtain the parallel wave number $k_\parallel$ as a function of ω for a given $k_\perp$ as follows:

$$k_\parallel^2 = A - \frac{k_\perp^2}{2} \pm \sqrt{\left(\frac{\omega}{\omega_{ci}}\right)^2 A^2 + \left(\frac{k_\perp^2}{2}\right)^2}. \tag{17}$$

where when $\omega/\omega_{ci} < 1$, the $+$ sign describes the shear Alfvén wave and the $-$ sign describes the compressional Alfvén wave. Since the magnitude of the parallel magnetic field fluctuation depends on the value of $k_\parallel$, the difference in the magnitude of the density fluctuation arises between the shear and compressional Alfvén waves.

3. Two-Fluid Theory

In this section, we briefly discuss the relationship between the density and magnetic field fluctuations based on the two-fluid theory for an infinitely uniform warm plasma. We assume that the temperature is spatially uniform and that the quasi-charge neutrality is well satisfied for the electron and ion density fluctuations, that is, $\tilde{n}_e = \tilde{n}_i (= \tilde{n})$. In this case, it is easier to study dynamics of the electron density fluctuation $\tilde{n}_e (= \tilde{n})$ rather than dynamics of the ion density fluctuation. Small perturbations of the electron density n_e and flow velocity $\boldsymbol{v}_e$ are described by the linearized two-fluid equations given by

$$\frac{\partial \tilde{n}_e}{\partial t} + \nabla \cdot (n_0 \tilde{\boldsymbol{v}}_e) = 0, \tag{18}$$

$$M_e \frac{\partial}{\partial t} \tilde{\boldsymbol{v}}_e = -e\tilde{\boldsymbol{E}} - e\tilde{\boldsymbol{v}}_e \times \boldsymbol{B}_0 - \frac{T_e}{n_0} \nabla \tilde{n}_e, \tag{19}$$

where T_e is the electron temperature. The perpendicular motion of the electron is well approximated by the $\boldsymbol{E} \times \boldsymbol{B}$ drift as

$$\tilde{\boldsymbol{v}}_{e\perp} \cong \frac{\tilde{\boldsymbol{E}} \times \boldsymbol{B}_0}{B_0^2}. \tag{20}$$

and the parallel electron motion is given by, from eq. (19),

$$\tilde{v}_{e\parallel} = -\mathrm{i}\frac{e\tilde{E}_\parallel}{M_e\omega} + \frac{k_\parallel T_e}{M_e\omega}\frac{\tilde{n}_e}{n_0}, \tag{21}$$

where $\tilde{E}_\parallel$ is the parallel component of the electric field fluctuation. Substituting eqs. (20) and (21) into eq. (18) and using eq. (4), we can obtain

$$\frac{\tilde{n}}{n_0} - \frac{\tilde{B}_\parallel}{B_0} = \frac{k_\parallel^2 v_{te}^2}{2\omega^2}\left[\frac{\tilde{n}}{n_0} - \mathrm{i}\frac{e\tilde{E}_\parallel}{k_\parallel T_e}\right], \tag{22}$$

where $v_{te} (= \sqrt{2T_e/M_e})$ is the electron thermal velocity. From eq. (22), we obtain two limiting relationships as

$$\frac{\tilde{n}}{n_0} = \frac{\tilde{B}_\parallel}{B_0}, \quad \text{for } \omega \gg |k_\parallel| v_{te}, \tag{23}$$

and

$$\frac{\tilde{n}}{n_0} = \mathrm{i}\frac{e\tilde{E}_\parallel}{k_\parallel T_e}, \quad \text{for } \omega \ll |k_\parallel| v_{te}. \tag{24}$$

Equation (23) is the same as eq. (10) obtained in the ideal MHD theory. Namely, when the wave phase velocity $\omega/|k_\parallel|$ is much higher than the electron thermal velocity, the ideal MHD result is reproduced. In the opposite limit, the density fluctuation is related to the parallel component $\tilde{E}_\parallel$ of the electric field fluctuation. We note that $\tilde{E}_\parallel = 0$ in the ideal MHD theory as shown in eq. (11). For electrostatic fluctuations described by $\tilde{\boldsymbol{E}} = -\nabla\tilde{\phi}$, ϕ being the electrostatic potential. Eq. (24) describes the Boltzmann relation given by

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\phi}}{T_e}. \tag{25}$$

In the intermediate region of the two limits, we have

$$\frac{\tilde{n}}{n_0} = \left(1 - \frac{k_\parallel^2 v_{te}^2}{2\omega^2}\right)^{-1}\left[\frac{\tilde{B}_\parallel}{B_0} - \mathrm{i}\frac{ek_\parallel \tilde{E}_\parallel}{M_e\omega^2}\right]. \tag{26}$$

We note that the parallel magnetic and electric fluctuations $\tilde{B}_\parallel$ and $\tilde{E}_\parallel$ are not independent and related to each other when the electromagnetic mode is specified, though eq. (26) shows that both $\tilde{B}_\parallel$ and $\tilde{E}_\parallel$ induce the density fluctuation. We see from eq. (26) that the density fluctuation diverges when $\omega^2 = k_\parallel^2 v_{te}^2/2$. This means the breakdown of the two-fluid theory, which is due to the lack of wave-particle resonance effects. Therefore, we need analyses based on the kinetic theory taking into account wave-particle resonance effects to obtain the relationship between the density and magnetic field fluctuations for the case of $\omega \sim |k_{\|}| v_{\rm te}$.

4. Kinetic Theory

In this section, we discuss the relationship between the density and magnetic field fluctuations based on the kinetic theory taking into account the wave-particle resonances. The starting point is the linearized Vlasov equation for the perturbed distribution function $\tilde{f}_{\rm e}$ of the electron given by

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla - \frac{e}{M_{\rm e} c} \boldsymbol{v} \times \boldsymbol{B}_0 \cdot \frac{\partial}{\partial \boldsymbol{v}}\right) \tilde{f}_{\rm e} = \frac{e}{M_{\rm e}} \left(\tilde{\boldsymbol{E}} + \frac{1}{c} \boldsymbol{v} \times \tilde{\boldsymbol{B}}\right) \cdot \frac{\partial}{\partial \boldsymbol{v}} F_0, \qquad (27)$$

where F_0 is the unperturbed distribution function of the electron. When the unperturbed distribution function F_0 is isotropic, the solution of eq. (27) in an infinitely uniform plasma is given by [2]

$$\tilde{f}_{\rm e} = {\rm i} \frac{e n_0}{M_{\rm e}} \sum_{n=-\infty}^{\infty} \frac{{\rm e}^{{\rm i}\xi \sin\phi - {\rm i}n\phi}}{\omega - n\omega_{\rm ce} - k_{\|} v_{\|}} \times \left\{ \tilde{E}_{\|} J_n(\xi) \frac{\partial F_0}{\partial v_{\|}} + \left[\frac{\boldsymbol{k}_\perp \cdot \tilde{\boldsymbol{E}}_\perp}{k_\perp} \frac{n\omega_{\rm ce}}{k_\perp} J_n(\xi) + {\rm i} \frac{\hat{z} \cdot (\boldsymbol{k}_\perp \times \tilde{\boldsymbol{E}}_\perp)}{k_\perp} v_\perp J'_n(\xi) \right] \frac{1}{v_\perp} \frac{\partial F_0}{\partial v_\perp} \right\}, \qquad (28)$$

where $\xi = k_\perp v_\perp / \omega_{\rm ce}$, $\omega_{\rm ce}$ $(= -eB_0/M_{\rm e})$ the electron cyclotron frequency, ϕ the angle between $\boldsymbol{v}_\perp$ and $\boldsymbol{k}_\perp$, $J_n(\xi)$ the Bessel function of order n and $J'_n = dJ_n/d\xi$. We assume here that the unperturbed distribution function F_0 is Maxwellian and is given by $F_0 = (\pi^{3/2} v_{\rm te}^3)^{-1} \exp(-v^2/v_{\rm te}^2)$. We also assume the quasi-charge neutrality, that is, $\tilde{n}_{\rm e} = \tilde{n}_{\rm i} (= \tilde{n})$. The density fluctuation has been calculated in ref. 3 for the case that the parallel component of the electric field fluctuation is neglected in eq. (28).

Integrating eq. (28) with respect to the velocity $\boldsymbol{v}$, we obtain

$$\tilde{n} = \int_0^\infty v_\perp dv_\perp \int_{-\infty}^\infty {\rm d}v_{\|} \int_0^{2\pi} {\rm d}\phi \, \tilde{f}_{\rm e} \cong \frac{e n_0}{M_{\rm e}} 2\pi \int_0^\infty v_\perp {\rm d}v_\perp \int_{-\infty}^\infty {\rm d}v_{\|} \left\{ -{\rm i} \frac{\boldsymbol{k}_\perp \cdot \tilde{\boldsymbol{E}}_\perp}{k_\perp} \frac{1}{k_\perp v_\perp} \frac{\partial F_0}{\partial v_\perp} \sum_{n=-\infty (n\neq 0)}^{\infty} J_n^2(\xi) + \frac{\hat{z} \cdot (\boldsymbol{k}_\perp \times \tilde{\boldsymbol{E}}_\perp)}{k_\perp} \frac{\partial F_0}{\partial v_\perp} \left[\frac{J_0(\xi) J_1(\xi)}{\omega - k_{\|} v_{\|}} + \sum_{n=-\infty (n\neq 0)}^{\infty} \frac{J_n(\xi) J'_n(\xi)}{n\omega_{\rm ce}} \right] + {\rm i} \tilde{E}_{\|} \frac{\partial F_0}{\partial v_{\|}} \times \left[\frac{J_0^2(\xi)}{\omega - k_{\|} v_{\|}} - \sum_{n=-\infty (n\neq 0)}^{\infty} \frac{J_n^2(\xi)}{n\omega_{\rm ce}} \right] \right\}, \qquad (29)$$

where we have used the approximation $1/(\omega - n\omega_{\rm ce} - k_{\|} v_{\|}) \cong -1/n\omega_{\rm ce}$ for $n \neq 0$ since $|\omega_{\rm ce}| \gg |\omega - k_{\|} v_{\|}|$. Using the following relations

$$\sum_{n=-\infty (n\neq 0)}^{\infty} \frac{J_n^2(\xi)}{n} = \sum_{n=-\infty (n\neq 0)}^{\infty} \frac{J_n(\xi) J'_n(\xi)}{n} = 0,$$

$$\sum_{n=-\infty (n\neq 0)}^{\infty} J_n^2(\xi) = 1 - J_0^2(\xi),$$

and also $J_0(\xi) \cong 1 - \xi^2/4$ for $|\xi| \ll 1$, we can obtain

$$\tilde{n} = \frac{e n_0}{M_{\rm e}} 2\pi \int_0^\infty v_\perp {\rm d}v_\perp \int_{-\infty}^\infty {\rm d}v_{\|} \left[-{\rm i} \frac{\boldsymbol{k}_\perp \cdot \tilde{\boldsymbol{E}}_\perp}{k_\perp} \frac{1}{\omega_{\rm ce}} \frac{k_\perp v_\perp}{2\omega_{\rm ce}} \frac{\partial F_0}{\partial v_\perp} + \frac{\hat{z} \cdot (\boldsymbol{k}_\perp \times \tilde{\boldsymbol{E}}_\perp)}{k_\perp} \frac{1}{\omega - k_{\|} v_{\|}} \frac{k_\perp v_\perp}{2\omega_{\rm ce}} \frac{\partial F_0}{\partial v_\perp} + {\rm i} \tilde{E}_{\|} \frac{1}{\omega - k_{\|} v_{\|}} \frac{\partial F_0}{\partial v_{\|}} \right], \qquad (30)$$

We see that the first term in eq. (30) can be neglected as compared with the second term, since the first term is smaller than the second term by $|\omega - k_{\|} v_{\|}| / |\omega_{\rm ce}|$ $(\ll 1)$. Using Faraday's law $\omega \tilde{\boldsymbol{B}} = \boldsymbol{k} \times \tilde{\boldsymbol{E}}$, we finally obtain the relationship between the density and magnetic field fluctuations as

$$\frac{\tilde{n}}{n_0} = -\frac{\omega}{|k_{\|}| v_{\rm te}} Z\left(\frac{\omega}{|k_{\|}| v_{\rm te}}\right) \frac{\tilde{B}_{\|}}{B_0} + {\rm i} \left[1 + \frac{\omega}{|k_{\|}| v_{\rm te}} Z\left(\frac{\omega}{|k_{\|}| v_{\rm te}}\right) \right] \frac{e \tilde{E}_{\|}}{k_{\|} T_{\rm e}}, \qquad (31)$$

where $Z(x)$ is the plasma dispersion function given by

$$Z(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{e^{-t^2}}{t - x} {\rm d}t. \qquad (32)$$

Equation (31) can reproduce the results of the two-fluid theory. That is, we obtain, from eq. (31),

$$\frac{\tilde{n}}{n_0} = \frac{\tilde{B}_{\|}}{B_0}, \quad \text{for } \omega \gg |k_{\|}| v_{\rm te},$$

and

$$\frac{\tilde{n}}{n_0} = \mathrm{i}\frac{e\tilde{E}_{\parallel}}{k_{\parallel}T_{\mathrm{e}}}, \text{ for } \omega \ll |k_{\parallel}|v_{\mathrm{te}}.$$

We note again that the parallel electric field fluctuation $\tilde{E}_{\parallel}$ is proportional to the parallel magnetic field fluctuation $\tilde{B}_{\parallel}$ in eq. (31) when the electromagnetic mode is specified. Equation (31) includes wave-particle resonance effects described by the imaginary part of the plasma dispersion function $Z(x)$. The wave-particle resonances describe the Landau damping or transit time damping by the electron. The imaginary contribution of $Z(x)$ due to wave-particle resonances causes the phase difference, for example, between the density fluctuation $\tilde{n}$ and the magnetic field fluctuation $\tilde{B}_{\parallel}$ as shown in ref. 3. On the other hand, the density and parallel magnetic field fluctuations in the two-fluid theory are in phase each other, since wave-particle resonances are neglected. We show $|xZ(x)|$ and $|1+xZ(x)|$ as a function of $x=\omega/|k_{\parallel}|v_{\mathrm{te}}$ in Fig. 1. We see that $|xZ(x)|\rightarrow 0$ and $|1+xZ(x)|\rightarrow 1$ as $x\rightarrow 0$, and also $|xZ(x)|\rightarrow 1$ and $|1+xZ(x)|\rightarrow 0$ as $x\rightarrow\infty$. We note that the imaginary part of $Z(x)$ becomes important for $x<2$, where the two-fluid theory breaks down.

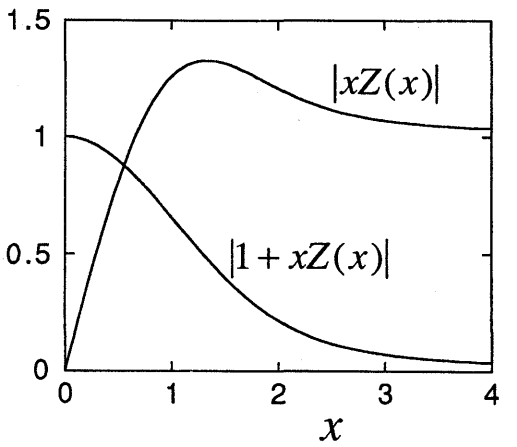

Fig. 1 $|xZ(x)|$ and $|1+xZ(x)|$ as a function of $x=\omega/|k_{\parallel}|v_{\mathrm{te}}$, where $Z(x)$ is the plasma dispersion function.

5. Alfvén Ion Cyclotron Fluctuations

In this section, we discuss the relationship between the density and magnetic field fluctuations for the AIC mode. We compare the expression (31) obtained by the kinetic theory in the previous section with the experimental results obtained by means of measurement using both O-mode and X-mode reflectometers in the GAMMA 10 experiment [4].

We first estimate the ratio of the density to magnetic field fluctuations theoretically. Therefore, we need to find the ratio of the parallel electric field to magnetic field fluctuations, i.e., $\tilde{E}_{\parallel}/\tilde{B}_{\parallel}$ for the AIC mode. Since the AIC mode is destabilized by the temperature anisotropy of the ion, we assume the bi-Maxwellian distribution function as the unperturbed distribution function of the ion, for simplicity, in order to estimate the ratio $\tilde{E}_{\parallel}/\tilde{B}_{\parallel}$. We also assume that $\tilde{E}_x=\mathrm{i}\tilde{E}_y$ since the AIC mode is left-hand polarized and $\boldsymbol{k}=k_{\perp}\hat{x}+k_{\parallel}\hat{z}$. In this case, from the dispersion equation of electromagnetic modes in an infinitely uniform magnetized plasma [5], we obtain,

$$[\varepsilon_{yz}-\mathrm{i}(\varepsilon_{xz}+n_{\perp}n_{\parallel})]\tilde{E}_y=(\varepsilon_{zz}-n_{\perp}^2)\tilde{E}_{\parallel}, \tag{33}$$

with $n_{\perp}=ck_{\perp}/\omega$, $n_{\parallel}=ck_{\parallel}/\omega$ and

$$\varepsilon_{xz}=-\frac{k_{\perp}}{k_{\parallel}}\frac{\omega_{\mathrm{pi}}^2}{\omega^2}\left\{\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}-1 +\frac{1}{2}\sum_{\lambda=\pm1}\left[\left(\lambda\frac{\omega}{\omega_{\mathrm{ci}}}+1\right)\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}-1\right] \times\frac{\omega+\lambda\omega_{\mathrm{ci}}}{|k_{\parallel}|v_{\mathrm{ti}}}Z\left(\frac{\omega+\lambda\omega_{\mathrm{ci}}}{|k_{\parallel}|v_{\mathrm{ti}}}\right)\right\}, \tag{34}$$

$$\varepsilon_{yz}=-\frac{k_{\perp}}{k_{\parallel}}\frac{\omega_{\mathrm{pi}}^2}{\omega^2}\left\{-\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}\frac{\omega}{|k_{\parallel}|v_{\mathrm{ti}}}Z\left(\frac{\omega}{|k_{\perp}|v_{\mathrm{ti}}}\right) +\frac{1}{2}\sum_{\lambda=\pm1}\left[\left(\lambda+\frac{\omega}{\omega_{\mathrm{ci}}}\right)\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}-\lambda\right] \times\frac{\omega+\lambda\omega_{\mathrm{ci}}}{|k_{\parallel}|v_{\mathrm{ti}}}Z\left(\frac{\omega+\lambda\omega_{\mathrm{ci}}}{|k_{\parallel}|v_{\mathrm{ti}}}\right)\right\}, \tag{35}$$

$$\varepsilon_{zz}=2\frac{\omega_{\mathrm{pe}}^2}{(k_{\parallel}v_{\mathrm{te}})^2}\left[1+\frac{\omega}{|k_{\parallel}|v_{\mathrm{te}}}Z\left(\frac{\omega}{|k_{\parallel}|v_{\mathrm{te}}}\right)\right], \tag{36}$$

where ε_{xz}, ε_{yz} and ε_{zz} are the elements of the dielectric tensor for the AIC mode, $T_{\perp\mathrm{i}}$ and $T_{\parallel\mathrm{i}}$ the perpendicular and parallel temperatures of the ion, respectively, $v_{\mathrm{ti}}=\sqrt{2T_{\parallel\mathrm{i}}/M_{\mathrm{i}}}$, $\omega_{\mathrm{p}j}\ (=\sqrt{e^2n_0/\varepsilon_0M_j}\,)$ the plasma frequency, and we have assumed $k_\perp\rho_{\mathrm{i}}\ll 1$ ($\rho_{\mathrm{i}}=\sqrt{T_{\perp\mathrm{i}}/M_{\mathrm{i}}}/\omega_{\mathrm{ci}}$). Using $\widetilde{E}_y=(\omega/k_\perp)\,\widetilde{\boldsymbol{B}}_\parallel$ and eq. (33), we then obtain,

$$\frac{\widetilde{E}_\parallel}{\widetilde{B}_\parallel}=\frac{\omega}{k_\perp}\,\frac{\varepsilon_{yz}-\mathrm{i}\,(\varepsilon_{xz}+n_\perp n_\parallel)}{\varepsilon_{zz}-n_\perp^2}. \tag{37}$$

If we substitute eqs. (34) through (36) into eq. (37), we can obtain, for $k_\perp\rho_{\mathrm{i}}\ll 1$,

$$\frac{\widetilde{E}_\parallel}{\widetilde{B}_\parallel}=\mathrm{i}\,\frac{v_{\mathrm{te}}}{2}\,\frac{M_{\mathrm{e}}}{M_{\mathrm{i}}}\,\frac{k_\parallel v_{\mathrm{te}}}{\omega}\times\frac{\left[\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}\left(1-\frac{\omega}{\omega_{\mathrm{ci}}}\right)-1\right]\left[1+\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{ti}}}Z\left(\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{ti}}}\right)\right]}{1+\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}Z\left(\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}\right)} \tag{38}$$

Since $eB_0/k_\parallel T_{\mathrm{e}}=2\,(M_{\mathrm{i}}/M_{\mathrm{e}})\,(\omega_{\mathrm{ci}}/k_\parallel v_{\mathrm{te}}^2)$, substituting eq. (38) into eq. (31), we finally obtain

$$\frac{\widetilde{n}}{n_0}=\left\{-\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}Z\left(\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}\right)-\frac{\omega_{\mathrm{ci}}}{\omega}\left[\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}\left(1-\frac{\omega}{\omega_{\mathrm{ci}}}\right)-1\right]\times\left[1+\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{ti}}}Z\left(\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{ti}}}\right)\right]\right\}\frac{\widetilde{B}_\parallel}{B_0}. \tag{39}$$

We next mention fluctuations which are considered to be the AIC modes observed in the GAMMA 10 experiment by means of the reflectometer measurement [4] and compare eq. (31) with the experimental results. The AIC modes have been already observed in GAMMA 10 by means of the magnetic probes, where the strong temperature anisotropy of the ion is created by ICRF heating in the central cell [6]. The fluctuations increase with the increase of the central-cell beta value and the temperature anisotropy of the ion, i. e., $T_{\perp\mathrm{i}}/T_{\parallel\mathrm{i}}$. The signals from both the O-mode and X-mode reflectometers can be consistently explained only by the existence of both the density and magnetic field fluctuations, since the O-mode signals are depending on the density fluctuation only and X-mode signals are related to both the density and magnetic field fluctuations. The ratio of the O-mode phase change $\delta\Phi_{\mathrm{O}}$ to X-mode phase change $\delta\Phi_{\mathrm{X}}$ is given by [7]

$$\frac{\delta\Phi_{\mathrm{O}}}{\delta\Phi_{\mathrm{X}}}=\frac{k_{\mathrm{O}}\,(\widetilde{n}_{\mathrm{e}}/n_0)}{k_{\mathrm{X}}\,[\,\widetilde{n}_{\mathrm{e}}/n_0+(\omega_{\mathrm{X}}\,|\,\omega_{\mathrm{ce}}\,|/\omega_{\mathrm{pe}}^2)\,\widetilde{B}_\parallel/B_0]}, \tag{40}$$

where ω_{X} is the frequency of the X-mode incident wave, k_{O} and k_{X} the wave numbers of the O-mode and X-mode incident waves, respectively. In the experiment, the ratio of the density to magnetic field fluctuations is estimated as $|\,\widetilde{n}_{\mathrm{e}}/n_0|/|\,\widetilde{B}_\parallel/B_0|=0.7-1.5$ from eq. (40). On the other hand, using the experimental results, $\omega\cong 0.9\omega_{\mathrm{ci}}$, $T_{\perp\mathrm{i}}/T_{\parallel\mathrm{i}}=8-14$, $\omega/|k_\parallel|\,v_{\mathrm{e}}=0.5-0.7$ and $(\omega-\omega_{\mathrm{ci}})/|k_\parallel|\,v_{\mathrm{ti}}=1.2\sim 2.0$, we obtain

$$\left|\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}Z\frac{\omega}{|k_\parallel|\,v_{\mathrm{te}}}\right|\approx 0.8-1.0,$$

$$\frac{\omega_{\mathrm{ci}}}{\omega}\left[\frac{T_{\perp\mathrm{i}}}{T_{\parallel\mathrm{i}}}\left(1-\frac{\omega}{\omega_{\mathrm{ci}}}\right)-1\right]\times\left|\left[1-\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{te}}}Z\left(\frac{\omega-\omega_{\mathrm{ci}}}{|k_\parallel|\,v_{\mathrm{te}}}\right)\right]\right|\approx-0.1\sim 0.2.$$

Then we obtain $|\,\widetilde{n}_{\mathrm{e}}/n_0|/|\,\widetilde{B}_\parallel/B_0|\approx 0.7-1.2$ from eq. (39) as the theoretical estimation. Therefore, we see that both theoretical and experimental results are in good agreement with each other. Since the second term is smaller than the first term by factor four to one order of magnitude in eq. (39), we also find that the parallel magnetic field fluctuation dominates the density fluctuation associated with the AIC modes observed in the GAMMA 10 experiment. This result justifies our previous analysis in ref. 3, where the contribution from the parallel electric field fluctuation is neglected. The density-fluctuation level is estimated as $|\,\widetilde{n}_{\mathrm{e}}/n_0|=(2-4)\times 10^{-4}$, which is very low as compared with the fluctuation level of electron drift waves, $|\,\widetilde{n}_{\mathrm{e}}/n_0|=(1-10)\times 10^{-2}$, observed in GAMMA 10 [8].

6. Summary

We have discussed the relationship between the density and magnetic field fluctuations associated with electromagnetic plasma waves in an infinitely uniform magnetized plasma based on the ideal MHD theory, two-fluid theory and kinetic theory. In the ideal MHD theory for a cold plasma, the density fluctuation is related to the parallel component of the magnetic field fluctuation. In the two-fluid theory for a warm plasma, where the quasi-charge neutrality is assumed, the density fluctuation arises from the parallel components of the magnetic and electric field fluctuations. When the wave phase velocity is much higher than the electron thermal velocity, the MHD result is reproduced. In the opposite limit, the Boltzmann relation is obtained for electrostatic fluctuations. The two-fluid theory breaks down when the wave phase velocity is comparable to the electron thermal velocity. This is due to the lack of wave-particle resonances. In the kinetic theory, we have obtained the expression for the density fluctuation including wave-particle resonances described in terms of the parallel components of the magnetic and electric field fluctuations. The kinetic expression covers the results of the two-fluid theory. Concerning with the ratio of the density to magnetic field fluctuations for the AIC modes observed in the GAMMA 10 experiment by means of the reflectometer measurement, we have compared the kinetic expression with the experimental results and have found that both results are in good agreement with each other.

Acknowledgments

The authors are grateful to the GAMMA 10 group of University of Tsukuba for useful discussions. This work was partly supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture.

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